INDEPENDENCE SCHOOL DISTRICT

Math Virtual Learning

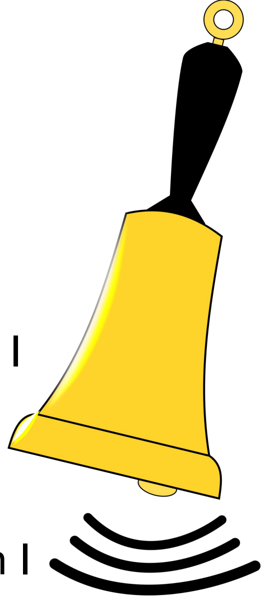
Probability and Statistics

May 4, 2020

Probability and Statistics
Lesson: May 4, 2020

Objective/Learning Target:
Students will be able to calculate the predicted range of a population mean with a given level (percent) of confidence

Let’s Get Started!



Use the [Z-score Conversion Chart] to find the z-scores for the given percentiles.

1. I am looking for the z-score that corresponds to the 83rd percentile. When I use the chart, my two options are 0.8289 and 0.8315. Which one should I choose?

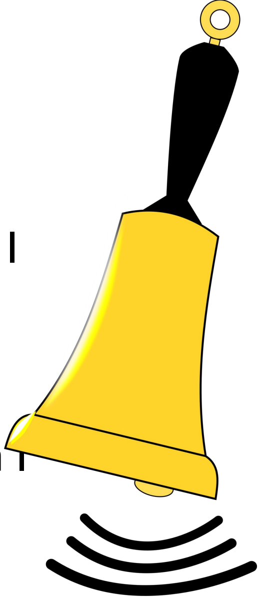
2. I am looking for the z-score that corresponds to the 42nd percentile. When I use the chart, my two options are 0.4168 and 0.4207. Which one should I choose?

3. What is the z-score for the 98th percentile?

4. What is the z-score for the 57th percentile?

5. What is the z-score for the 38th percentile?

Let's Get Started! ANSWERS



1. I am looking for the z-score that corresponds to the 83rd percentile. When I use the chart, my two options are 0.8289 and 0.8315. Which one should I choose? 0.8289 -- 82.89 is closer to 83 (0.11 away) than 83.15 is (0.15 away)

2. I am looking for the z-score that corresponds to the 42nd percentile. When I use the chart, my two options are 0.4168 and 0.4207. Which one should I choose? 0.4207 -- 42.07 is closer to 42 (0.07 away) than 41.68 is (0.32 away)

3. What is the z-score for the 98th percentile? 2.05

4. What is the z-score for the 57th percentile? 0.18

5. What is the z-score for the 38th percentile? -0.31

Confidence Intervals

So on our last lessons we practiced finding the Standard Deviation of a Sample when we know the Population Mean and Population Standard Deviation. Then we used that information to answer questions about data points and ranges of data points.

But what happens if you don't know the Population Mean and Standard Deviation? After all, how often can you really survey, measure, weigh, test,...**EVERY SINGLE PERSON** in a population? *(Not very likely)*

Now how are you going to get the values you need if you don't have the starting information?

*(Some of the information I will be using today comes from this **Website**, feel free to look at it yourself for reference and graphics.)*

Let’s look at an example...

Suppose we want to know the Population Mean (called the “True Mean”) of all men. Well we know that it is impossible to measure **every single man** that lives to get true Population statistics!

We said in the last lesson that the population mean and the sample mean are the same, which they are…...give or take a little bit of wiggle room.

We call that wiggle room a **Margin of Error** (which you’ll learn to calculate in slide 10)

Confidence Intervals

Using that **Margin of Error**, we are able to build something called a **Confidence Interval** where we calculate a range of heights and can be sure with level of confidence that the True Mean falls between these 2 points. Sometimes we may want to 95% Confidence. Other times we may only need to be 80% Confident. Other times we may want to be almost absolutely sure and say that we are 99% Confident. *The degree of which we want to be Confident will be told in the problem.*

NOTE: Because we don't measure the ENTIRE POPULATION, we can never be 100% confident of our range...unless we do something outrageous like 0-10000000 which is not helpful at all statistically. But we can get pretty close and have a 99.9% confidence.

Confidence Intervals in Action - Example 1

Dilemma: We need the True Mean (population mean) of the height of <u>all men.</u>

We can't reasonably measure all men in the world. So we start by taking a sample, let's say we measure the heights of 40 random men. From that sample, we find that the Sample Mean = 175 cm and the Sample Standard Deviation = 20 cm.

Since we are going from Sample backwards to Population, we also have to go backwards from [% to Z-Score] for these.

However, you will be given a chart of the most popular Confidence Intervals so that the hard part of working backwards by looking up percentages in the Z-Score Chart is done for you. YAY!!!

Popular Confidence Ranges

Here is the Chart that you will need of the 7 most popular confidence ranges:

(if you want a Confidence Interval other than these 7 listed, you will have to use the Z-Score Chart and work backwards like we did in previous lessons)

Confidence Interval	Z
80%	1.282
85%	1.440
90%	1.645
95%	1.960
99%	2.576
99.5%	2.807
99.9%	3.291

Confidence Intervals in Action Example #1 Continued...

So we start by taking a sample, let's say we measure the heights of 40 random men. From that sample, we find that the Sample Mean = 175 cm and the Sample Standard Deviation = 20 cm.

Let's say that we want a Confidence Interval of <u>95%</u> for this problem. This means that we want to be 95% sure that the true mean is in the range we say it is.

Use this formula to find the "Margin of Error"

$$Z\frac{s}{\sqrt{n}} \qquad 1.960 * \frac{20}{\sqrt{40}} = 6.20\ cm$$

Z-Score for 95% from Chart (previous slide) = 1.960

(notice that it is VERY similar to finding the Sample Standard Deviation except that there is an extra step. You must multiply by the Z-Score from the chart on the previous slide)

Margin of Error to Confidence Interval

Our Margin of Error is 6.20 cm (this is what we call our “wiggle room”)

Sample Mean: 175

Add and Subtract the Margin of Error of each side of the Sample mean like this:

175 - 6.20 , 175 + 6.20
168.8 , 181.2

****Be careful NOT to confuse this answer with Standard Deviation! It looks just like how we did Standard Dev but THIS is called* ***Margin of Error****!*

So now I can say that I am 95% Confident that the True Mean (population) lies somewhere between 168.8 cm and 181.2 cm.

Confidence Intervals in Action - Example #2

Dilemma: We need to know if the apples in our Orchard are big enough to sell. It would be a waste to test them all because then there would be none to sell (plus it would take too much time). Find the True mean with 90% confidence.

So we start by taking a sample, let's say we measure the weight of 46 random apples in the orchard. From that sample, we find that the Sample Mean = 86 g and the Sample Standard Deviation = 6.2 g.

Confidence Intervals in Action - Example #2 Cont.

Dilemma: We need to know if the apples in our Orchard are big enough to sell. It would be a waste to test them all because then there would be none to sell (plus it would take too much time). Find the True mean with 90% confidence.

So we start by taking a sample, let's say we measure the weight of 46 random apples in the orchard. From that sample, we find that the Sample Mean = 86 g and the Sample Standard Deviation = 6.2 g.

90% confidence is a z-score of 1.645. Therefore our wiggle room number is 1.50

$$1.645 * \frac{6.2}{\sqrt{46}} = 1.50$$

Confidence Intervals in Action - Example #2 Cont.

Dilemma: We need to know if the apples in our Orchard are big enough to sell. It would be a waste to test them all because then there would be none to sell (plus it would take too much time). Find the True mean with 90% confidence.

So we start by taking a sample, let's say we measure the weight of 46 random apples in the orchard. From that sample, we find that the Sample Mean = 86 g and the Sample Standard Deviation = 6.2 g.

90% confidence is a z-score of 1.645. Therefore our wiggle room number is 1.50 grams

86 - 1.50 = 84.5 86 + 1.50 = 87.5

Therefore, you can be 90% confidence that the entire population of apples in the orchard have a mean weight between 84.5 - 87.5 grams.

You Try One!

Health Professionals use the term Hazard Ratio to state the health risk level of adults. A measure 1 is “normal”. A score below a 1 means that they are at LESS risk and a score above 1 means they are at MORE risk. A research hospital would like to know the mean Hazard Ratio of women 30 and older in the their city. 1350 women were sampled and found to have sample mean of 0.96 (slightly less than normal risk) with a standard deviation of 0.89.

State, with 99% confidence, the mean Hazard Ratio for the entire population of women in this city.

You Try One! ANSWERS

Health Professionals use the term Hazard Ratio to state the health risk level of adults. A measure 1 is "normal". A score below a 1 means that they are at LESS risk and a score above 1 means they are at MORE risk. A research hospital would like to know the mean Hazard Ratio of women 30 and older in the their city. 1350 women were sampled and found to have sample mean of 0.96 (slightly less risk) with a standard deviation of 0.89.

State, with 99% confidence, the mean Hazard Ratio for the entire population of women in this city.

99% confidence has a z-score of 2.576

$$2.576 * \frac{0.89}{\sqrt{1350}} = 0.06$$

0.96 - 0.06 = 0.90
0.96 + 0.06 = 1.02

You can be 99% confident that the mean Hazard Ratio of women in this city is between 0.90 and 1.02

Now Try Another!

A news article recently published that the mean SAT math score for a group of 50 students who took the SAT last year was 600. This sample had a standard deviation of 50. Find, with 95% confidence, the mean SAT math score of all students in the United States who took the SAT last year.

Now Try Another!

A news article recently published that the mean SAT math score for a group of 100 students who took the SAT last year was 600. This sample had a standard deviation of 50. Find, with 95% confidence, the mean SAT math score of all students in the United States who took the SAT last year.

95% has a z-score of 1.96

$$1.96 * \frac{50}{\sqrt{100}} = 9.8$$

600 - 9.8 = 590.2
600 + 9.8 = 609.8

You can be 95% confident that the mean math score for students who took the SAT last year is between 590.2 and 609.8